AIAA JOURNAL
Vol. 44, No. 1, January 2006

Comparative Scaling of Flapping- and Fixed-Wing Flyers

Tianshu Liu*
Western Michigan University, Kalamazoo, Michigan 49008

The scaling laws in the geometry, velocity, and power for flapping flyers, for example, birds, and fixed-wing aircraft are discussed from a comparative point of view, and the aerodynamic implications of the scaling, particularly on the lift-to-drag ratio, flapping span efficiency, induced drag, parasite drag, and propulsive efficiency are explored. The results shed insights into flapping flight and provide a useful guideline for the preliminary design of a flapping-flight vehicle.

I. Introduction

PEOPLE, who have always been amazed by biological flapping flight, have tried to make mechanized flapping air vehicles for a long time. The early design of a flapping-flight machine inspired by bird flight can be traced back to Leonardo da Vinci in the 1500s. Because of the formidable challenge of replicating bird flight and the failure to produce a workable man-carrying ornithopter, however, the idea of flapping flight was largely abandoned by mainstream aerospace engineers as a feasible engineering solution for flight, particularly after the Wright brothers achieved the first powered flight with a fixed-wing aircraft, followed by the remarkable progress of modern aviation made in the 20th century. As a result, flapping flight becomes a marginalized topic that mainly interests avian biologists, zoologists, and a few ornithopter inventors, although aerodynamicists, such as von Kármán and Burgers,[1] Garrick,[2] von Holst and Kuchemann,[3] and Lighthill,[4,5] have embarked on this topic. Recently, there is renewed interest to flapping flight in the aerospace community due to the need of developing birdlike micro air vehicles. In regard to flapping flight, a long-standing problem is the relative efficiency of flapping flight vs fixed-wing flight.

In this paper, flapping- and fixed-wing flyers are compared from a standpoint of scaling. Scaling is a useful technique for biologists and aerodynamicists to study the dimensional relationship between different scales and species in avian flight.[4,6–10] Flying animals (birds, bats, and insects) typically utilize flapping wings to generate both the thrust and lift for flight in contrast to manufactured fixed-wing aircraft that use propellers and jets for propulsion. A question is whether there is a statistically significant difference in the scaling laws between birds and aircraft that may be caused by the two different flight mechanisms: flapping wings and fixed wings with a propulsion engine. If there is a difference, what are its aerodynamic implications? The aerodynamic aspects of the scaling are very worthwhile to be explored. In addition, the scaling laws provide a useful guideline for the preliminary design of flapping MAVs or ornithopters and lay a foundation for more detailed aerodynamic research of flapping flight.

II. Comparative Scaling

Weight, Lengths, and Area

Traditionally, biologists scale the relevant parameters of birds based on the mass m of birds. In engineering, the weight $W = mg$ is often used. Aircraft have two characteristic weights: the empty weight W_{empty} and the maximum takeoff (MTO) weight W_{MTO}. As shown in Fig. 1, the MTO weight is related to the empty weight by $W_{\mathrm{MTO}} = 1.786 W_{\mathrm{empty}}$ (12%) for propeller/turboprop aircraft and jet transports, where both W_{MTO} and W_{empty} are in newtons. As indicated in the parentheses, the mean relative error for regression is about 12%. In the following discussions, the mean aircraft weight $W = (W_{\mathrm{MTO}} + W_{\mathrm{empty}})/2 = 1.393 W_{\mathrm{empty}}$, which is simply referred to as the weight, is used for scaling, although W_{MTO} is also used in certain cases. Thus, a relation for aircraft is $W_{\mathrm{MTO}} = 1.282 W$ (12%). For birds, W is also referred to as the mean bird weight. The weight of birds varies depending on day, season, sex, and geographic locations, and such a variation can be described by the upper and lower bounds of the bird weight.[11] As shown in Fig. 2, the upper bound of the bird weight is related to the mean weight by $W_{\mathrm{upper,bird}} = 1.242 W$ (9%), which is interestingly close to the relation $W_{\mathrm{MTO}} = 1.282 W$ for aircraft. Similarly, the lower bound of the bird weight is given by $W_{\mathrm{low,bird}} = 0.79 W$ (10%). For geometrically similar objects, the linear length scale l is proportional to the one-third-power of the weight W, that is,

$$l \sim W^{\frac{1}{3}} \tag{1}$$

Therefore, the wingspan b, mean wing chord $\bar{c}$, overall body length l_{body}, and maximum diameter d_0 of an aircraft fuselage (or body) should be proportional to $W^{1/3}$, whereas the wing area follows $S_{\mathrm{wing}} \sim W^{2/3}$. Here, the wing area S_{wing} is defined as the orthographically projected area of a wing.

Figures 3 and 4 show the wingspan and wing area as a function of the weight for birds and aircraft, respectively. We use data collected by Tennekes[12] for birds ranging from a 0.026-N black-chinned hummingbird to a 116-N mute swan. Data for propeller/turboprop aircraft and jet transports published in Ref. 13 are used, covering a broad spectrum of aircraft from a 150-kg ultralight to a 180-ton Boeing 747-400. The results indicate that, over a large range of the weight, birds and aircraft basically follow the power laws for the wingspan and wing area. For the wingspan, we have for aircraft (14%) and birds (20%), respectively,

$$b = 0.462 W^{\frac{1}{3}} \tag{2}$$

$$b = 0.506 W^{\frac{1}{3}} \tag{3}$$

For the wing area, the scaling laws are for aircraft (30%) and birds (34%), respectively,

$$S_{\mathrm{wing}} = 0.0262 W^{\frac{2}{3}} \tag{4}$$

$$S_{\mathrm{wing}} = 0.0327 W^{\frac{2}{3}} \tag{5}$$

where b is in meters, S_{wing} is in square meters, and W is in newtons. In this work, we generally use the SI units such as meters for length scale, meters per second for velocity, newtons for weight, and watts for power, unless specified otherwise. For each empirical scaling law given in this paper, the mean relative error of regression is indicated accordingly in a parenthesis. For example, the mean relative error

Received 22 July 2003; revision received 8 March 2005; accepted for publication 25 June 2005.

*Associate Professor, Department of Mechanical and Aeronautical Engineering, G-220, Parkview Campus; tianshu.liu@wmich.edu. Member AIAA.

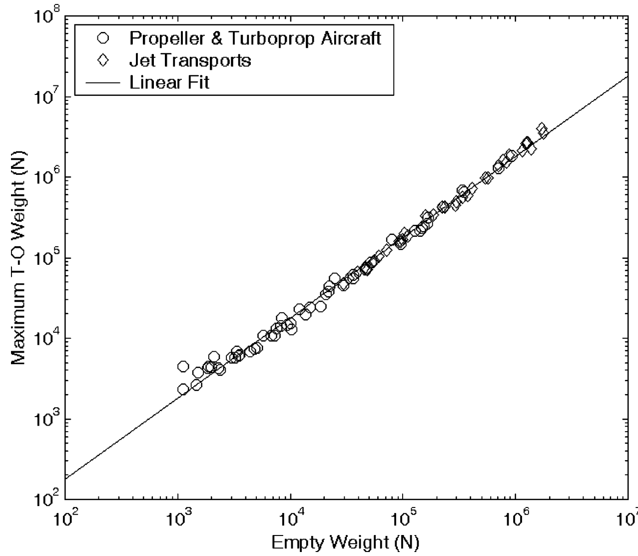

Fig. 1 MTO weight as function of empty weight for aircraft.

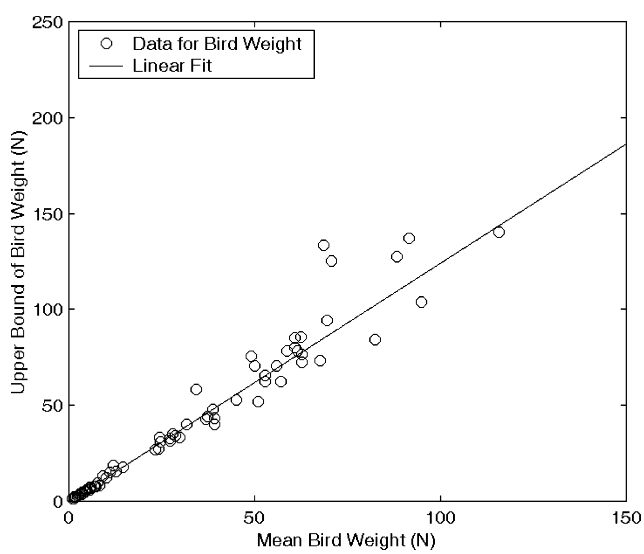

Fig. 2 Upper bound of bird weight as function of mean bird weight.

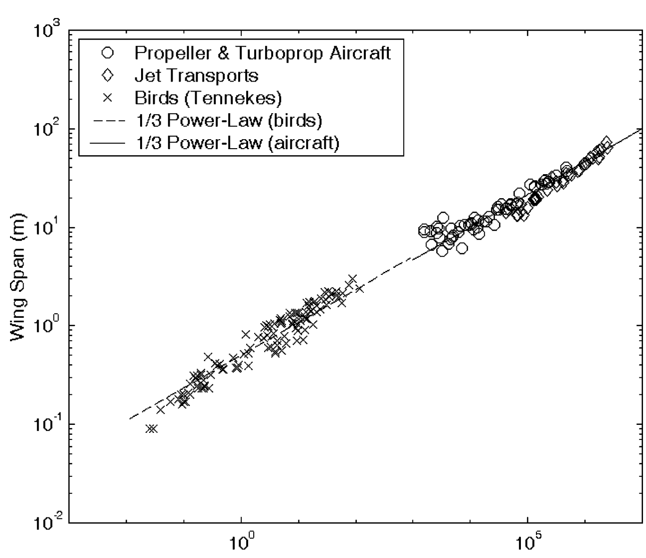

Fig. 3 Wingspan as function of weight.

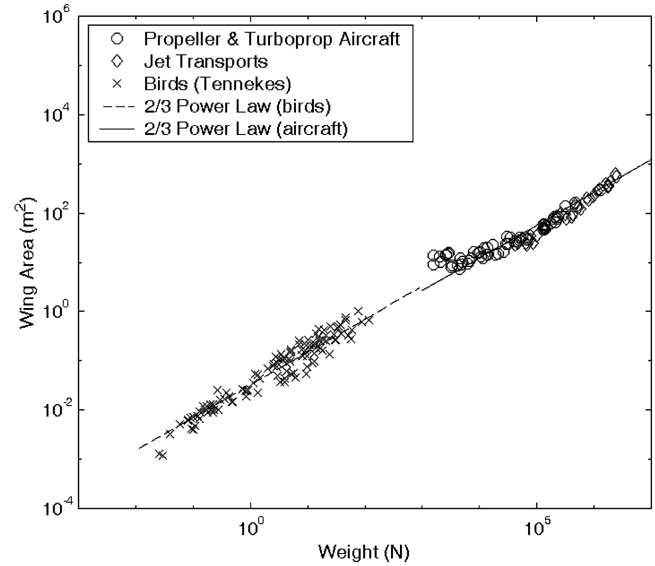

Fig. 4 Wing area as function of weight.

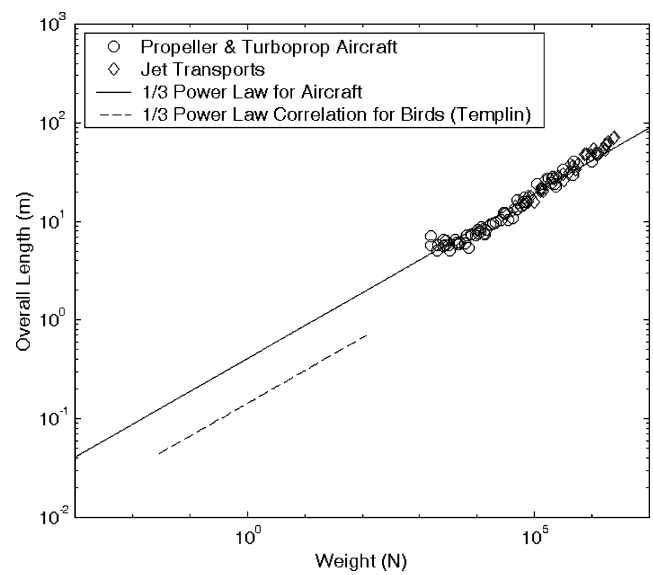

Fig. 5 Overall body length as function of weight.

for the wingspan is defined as mean($|b - b_{\mathrm{fit}}|/|b|$) over the whole range of data.

Note that a finer analysis given by Greenewalt[6] shows some variation in the scaling of the wingspan and wing area for different groups of bird species, such as the passeriforms, shorebirds and ducks. However, in a coarser scale considered here, the subtle difference between bird species is not statistically significant. McMasters[14] gave a similar scaling law $S_{\mathrm{wing}} = 0.0359 W^{2/3}$ for insects, birds, sailplanes, general aviation aircraft, and jet transports. Because the mean wing chord is defined as $\bar{c} = S_{\mathrm{wing}}/b$, Eqs. (2–5) lead to the following relations for aircraft (33%) and birds (39%):

$$\bar{c} = 0.0567\ W^{\frac{1}{3}} \tag{6}$$

$$\bar{c} = 0.0646\ W^{\frac{1}{3}} \tag{7}$$

Furthermore, from Eqs. (2–5), we know that the average wing aspect ratios $\mathcal{R} = b^2/S_{\mathrm{wing}}$ for aircraft and birds are 8.15 (33%) and 7.83 (39%), respectively, where the relative errors are estimated using the error propagation equation.

Figure 5 shows the overall body length as a function of the weight for aircraft along with a correlation for birds $l_{\mathrm{body}} = 0.143 W^{1/3}$ given by Templin.[15] For aircraft (12%), the overall length of aircraft obeys

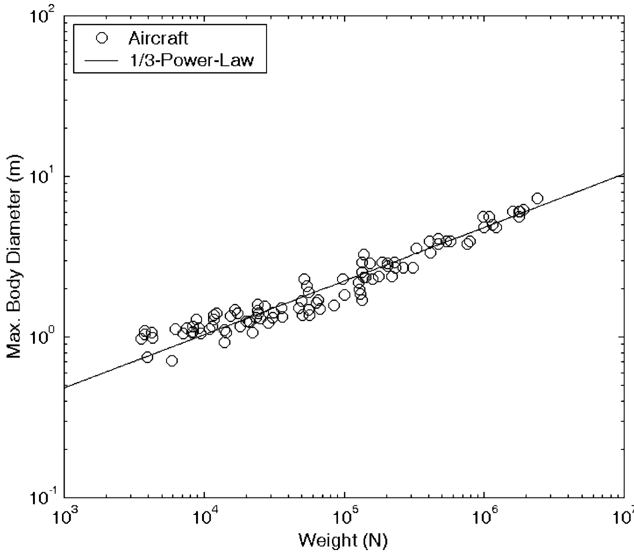

Fig. 6 Maximum diameter of fuselage or body as function of weight.

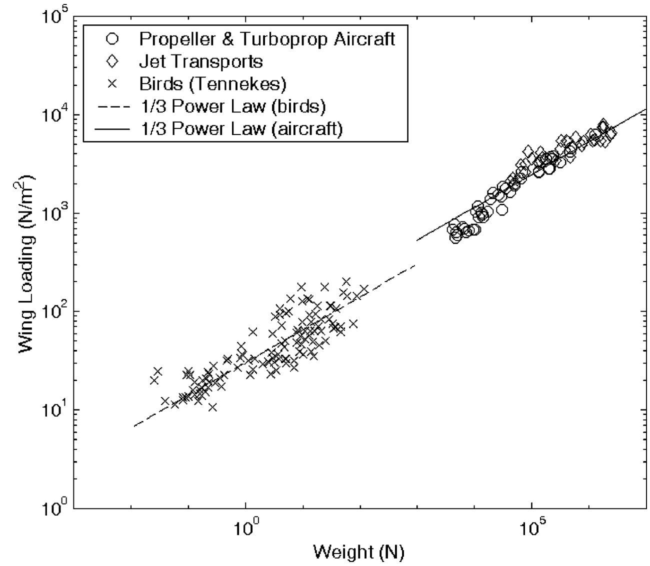

Fig. 7 Wing loading as function of weight.

the power law

$$l_{\text{body}} = 0.41 W^{\frac{1}{3}} \tag{8}$$

The scaling indicates that a typical bird has about 35% of the overall body length of the corresponding scaled-down aircraft. As shown in Fig. 6, the maximum diameter d_0 of the aircraft fuselage (or body) also follows the one-third-power law as a function of W given by

$$d_0 = 0.0481 W^{\frac{1}{3}} \tag{9}$$

for aircraft (13%). If the total area of the fuselage plus other parts of aircraft except the wing is approximately equivalent to the peripheral area of a cylinder of the diameter d_0 and the length l_{body}, the total wet area of a typical aircraft is $S_{\text{wet}} \approx 0.114 W^{2/3}$, and thus a ratio between the wing area and the total wet area of aircraft is $S_{\text{wet}}/S_{\text{wing}} \approx 4.35$. Unfortunately, data for the maximum diameter d_0 of the bird body are not available for comparison. When it is assumed that Eq. (9) can be approximately applied to birds, we have an estimate for birds $S_{\text{wet}} \approx 0.087 W^{2/3}$ and a ratio between the total wet area and the wing area for birds is $S_{\text{wet}}/S_{\text{wing}} \approx 2.66$. The estimated fineness ratios l_{body}/d_0 for aircraft and birds are approximately 8.52 and 2.97, respectively.

Wing Loading and Cruise Velocity

Figure 7 shows the scaling for the wing loading for aircraft (20%) and birds (32%) that follows:

$$\text{wing loading} = 53 W^{\frac{1}{3}} \tag{10}$$

$$\text{wing loading} = 30.6 W^{\frac{1}{3}} \tag{11}$$

For level forward flight, the wing loading W/S_{wing} is related to the flight velocity V by $W/S_{\text{wing}} = \rho V^2 C_L/2$, where C_L is the lift coefficient that is a function of the angle of attack. According to Eq. (10) or (11), the cruise velocity, which corresponds to the maximum lift-to-drag ratio or the maximum range, follows the scaling law $V_{\text{mr}} \sim W^{1/6}$. Figure 8 shows the cruise velocity as a function of the weight for birds and aircraft. Tennekes[12] called this kind of plot "The Great Flight Diagram" and gave a loose single scaling law. In fact, there is a significant statistical difference of the cruise velocity data between birds, propeller/turboprop aircraft, and jet transports. Data for propeller/turboprop aircraft (28%) and birds (16%) are, respectively, fitted by

$$V_{\text{mr}} = 15.52 W^{\frac{1}{6}} \tag{12}$$

$$V_{\text{mr}} = 8.98 W^{\frac{1}{6}} \tag{13}$$

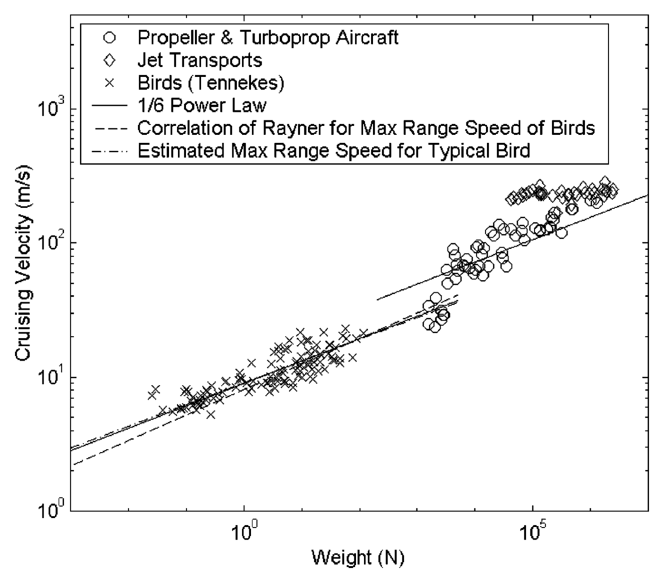

Fig. 8 Cruise velocity as function of weight.

where V_{mr} is in meters per second and W is in newtons. Nevertheless, the cruise velocity for jet transports does not significantly change with the weight, indicating that jets are not analogous to propeller/turboprop aircraft and birds in this aspect. For comparison, Fig. 8 also includes Rayner's correlation for the cruise velocity $V_{\text{mr}} = 8.0952 W^{0.19}$ for birds,[16] and the cruise velocity $V_{\text{mr}} = 9.067 W^{0.162}$ given for a typical bird in Sec. III based on the time-area-averaged momentum stream tube (MST) theory where the flapping span efficiency $e_{\text{flap}} = 0.5$ and the parasite (zero-lift) drag coefficient $C_{D_{\text{para}}} = 0.0055$ are chosen to fit both the cruise velocity and power data for birds. These results are consistent with the Tennekes data for birds. It is found that the cruise velocity of birds is considerably lower than that of scaled-down propeller/turboprop aircraft, that is, $V_{\text{mr,bird}} \approx 0.58 V_{\text{mr,aircraft}}$. This result is critical because it will affect estimation of several derived quantities such as the induced drag and propulsive efficiency. As pointed out in Sec. III, the scaling law for the cruise velocity is particularly susceptible to the deviations from the similarity conditions, and thus, the data look scattered because the functional dependency of the velocity to the weight is weak. Even after the error margins are considered, nevertheless, it is statistically credible that the cruise velocity of birds is significantly lower than that of scaled-down propeller/turboprop aircraft.

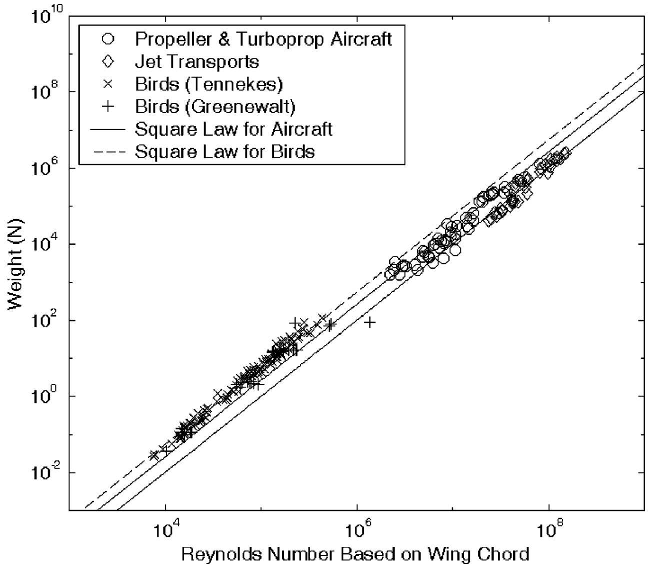

Fig. 9 Weight as function of Reynolds number.

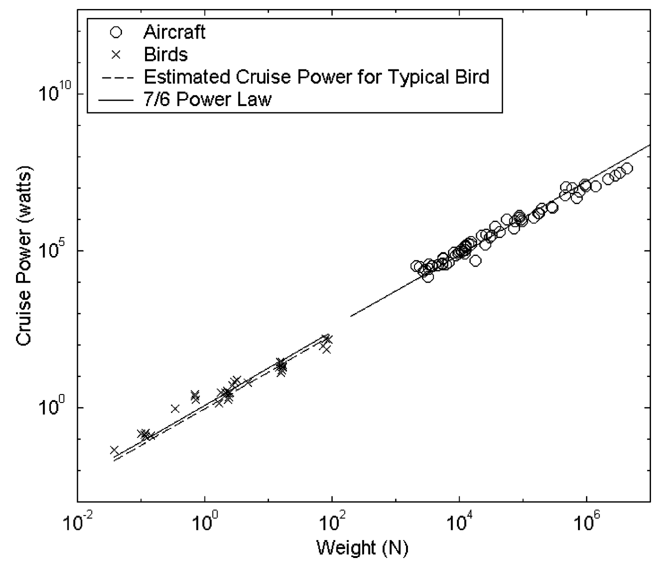

Fig. 10 Cruise power as function of weight.

We look at the relationship between the weight and the Reynolds number $Re_c = V\bar{c}/\nu$ based on the mean wing chord $\bar{c} = S_{\text{wing}}/b$. Figure 9 shows the weight of birds and aircraft as a function of the Reynolds number Re_c. Clearly, birds and aircraft have different square-laws.

Propeller/turboprop aircraft (52%):

$$W = 2.6 \times 10^{-10} Re_c^2 \tag{14}$$

Jet transports (31%):

$$W = 0.99 \times 10^{-10} Re_c^2 \tag{15}$$

Birds (32%):

$$W = 5.4 \times 10^{-10} Re_c^2 \tag{16}$$

Interestingly, this indicates that a typical flying bird is heavier than a scaled-down fixed-wing aircraft at the same Reynolds number. In the scaling sense, the mean weight of birds is about 2.6 times of that of scaled-down propeller/turboprop aircraft. In terms of the MTO weight of propeller/turboprop aircraft, we know $W_{\text{bird}} = 2.07 W_{\text{aircraft}} = 1.61 W_{\text{MTO}}$, and thus, the upper bound of the bird weight is $W_{\text{upper,bird}} = 2W_{\text{MTO}}$. Relatively speaking, this implies that a bird has to produce the larger lift to support its heavier body.

Cruise Power and Power Available

The mechanical power required for continuous flight is an important parameter to evaluate the flight performance. The maximum range power (the cruise power) P_{mr} at cruising is a product of the thrust T and the cruise velocity V_{mr}, that is, $P_{\text{mr}} = T V_{\text{mr}} = D V_{\text{mr}}$ in level continuous flight where the thrust is balanced by the drag D, and the lift is balanced by the weight. For a given lift-to-drag ratio, the thrust is proportional to the weight. Because the cruising velocity is $V_{\text{mr}} \sim W^{1/6}$, we know $P_{\text{mr}} \sim W^{7/6}$. Figure 10 shows the cruise power of aircraft and birds. The scaling laws for the cruise power of aircraft (38%) and birds (47%) are, respectively,

$$P_{\text{mr,aircraft}} = 1.67 W^{\frac{7}{6}} \tag{17}$$

$$P_{\text{mr,bird}} = 1.23 W^{\frac{7}{6}} \tag{18}$$

where the power is in watts and W is in newtons. Comparing Eqs. (17) with (18), we have $P_{\text{mr,bird}} \approx 0.74 P_{\text{mr,aircraft}}$, indicating that birds use less power for cruising flight compared to scaled-down aircraft. We collect data of the minimum power P_{mp} for birds from literature[6,17–21] and then convert them to the maximum range power P_{mr} using the relation $P_{\text{mr}} = 1.146 P_{\text{mp}}$ (see Sec. III). Except for the data of the mechanical power given by Greenewalt[6] and Dial et al.,[17] the metabolic power from other sources is converted to the mechanical power by multiplying 20%. As noted by Rayner,[21] the converting factor from the metabolic to mechanical power varies considerably. Therefore, the factor of 20% is a rough estimate. In Fig. 10, some data of the cruise power for propeller-driven aircraft are approximately extracted from the maximum rate of climb given in Ref. 13 (with the error of estimation typically less than 10%), and other data are collected from alternative sources. Figure 10 also shows the cruise power $P_{\text{mr}} = 0.9542 W^{1.161}$ obtained for a typical bird discussed in Sec. III by fitting the calculated data based on the MST theory where the flapping span efficiency $e_{\text{flap}} = 0.5$ and the parasite (zero-lift) drag coefficient $C_{D_{\text{para}}} = 0.0055$ are selected to fit both the cruise velocity and power data. This result is close to a correlation $P_{\text{mr,bird}} = 1.0564 W^{1.161}$ given by Rayner[16] and is in reasonable agreement with the collected data for birds. Although the power-law exponent can be generally treated as a free parameter for fitting data of the required power, it is preferred to fix the exponent at $\frac{7}{6}$ due to the rigorous constraint of aerodynamic similarity (see Sec. III).

However, the cruise power alone does not tell the whole story because the cruise velocity of birds is much lower than that of scaled-down aircraft. We need to evaluate the flight efficiency $\eta_{\text{flight}} = V_{\text{mr}} W / P_{\text{mr}}$ using the power laws. For propeller/turboprop aircraft, a value of η_{flight} based on the mean weight is $\eta_{\text{flight}} = V_{\text{mr}} W / P_{\text{mr}} \approx 9.3$, and hence, the mean lift-to-drag ratio is $L/D \approx 9.3$. To estimate the maximum flight efficiency, the MTO weight $W_{\text{MTO}} = 1.282 W$ is a more appropriate quantity; thus, we have $\eta_{\text{flight,max}} = V_{\text{mr}} W_{\text{MTO}} / P_{\text{mr}} \approx 11.9$. Because $\eta_{\text{flight,max}}$ is approximately equal to the maximum lift-to-drag ratio, we know $(L/D)_{\text{max}} \approx 11.9$ for propeller/turboprop aircraft. Similarly, the average flight efficiency for birds is $\eta_{\text{flight}} = V_{\text{mr}} W / P_{\text{mr}} \approx 7.3$, and therefore, the mean lift-to-drag ratio $L/D \approx 7.3$ for birds. Using the upper bound of the bird weight $W_{\text{upper,bird}} = 1.242 W$ for the maximum lift-to-drag ratio, we give an estimate $(L/D)_{\text{max}} \approx 9.1$ for birds. Hence, the maximum lift-to-drag ratio of birds is smaller, but close to that of propeller/turboprop aircraft. From Eqs. (14) and (16), compared to scaled-down propeller/turboprop aircraft at the same Reynolds numbers, an estimate for the mean lift coefficient of birds is $C_{L,\text{bird}} / C_{L,\text{aircraft}} \approx W_{\text{bird}} / W_{\text{aircraft}} = 2.07$. As a result, an estimate for the drag coefficient at the maximum L/D is $C_{D,\text{bird}} / C_{D,\text{aircraft}} \approx 2.7$. It is a somewhat surprising finding that the total drag coefficient of birds is much larger than that of propeller/turboprop aircraft. A detailed discussion on the induced drag and parasite drag is given in Sec. III. The larger lift coefficient of bird flight is worthwhile to be studied and is related to nonlinear interaction between the wing motion and the induced velocity by the vortex structure of an unsteady wake.

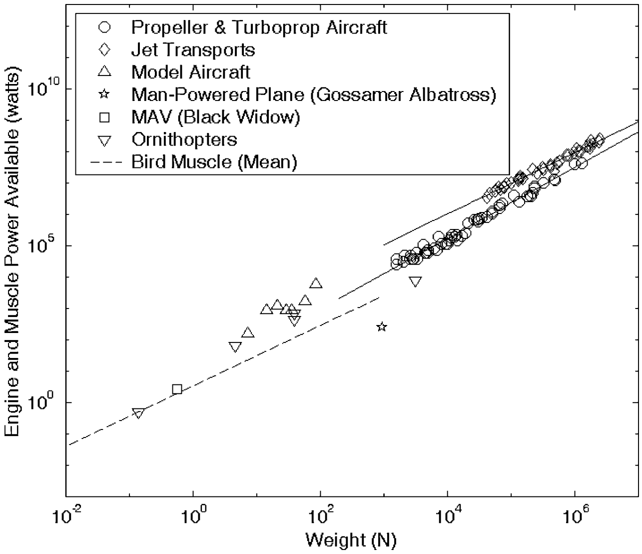

Fig. 11 Engine and muscle power available as function weight.

Figure 11 shows the engine and muscle power available as a function of the weight for aircraft and birds. For jet transports, the engine power is estimated by multiplying the maximum thrust to the cruise velocity at the corresponding cruising altitude. (Data are from Ref. 13.) Power data for model airplanes are collected from the specification sheets of SIG Manufacturing Co., Inc. (www.sigmfg.com [cited 15 October 2002]). A typical MAV, the Black Widow,[22] is used for comparison with birds. For the Black Widow that is driven by an electric motor powered by batteries, the mechanical power is estimated by multiplying the motor efficiency of 63% to the motor power. A number of ornithopters are also provided for comparison, including small-scale and full-scale ornithopters[23,24] and Microbat of AeroVironment.[25] The man-powered plane Gossamer Albatross[12] is marked in Fig. 11 as a unique reference case.

As indicated in the next subsection, the averaged weight of the large pectoral muscle of birds is $W_{\mathrm{muscle}} = 0.2363W^{0.9675}$, and the mean mass-specific power is 141 W/kg (14.4 W/N). Thus, the mean muscle power available for birds is $P_{A,\mathrm{muscle}} = 3.4W^{0.9675}$, which is also shown in Fig. 11. Note that a plot similar to Fig. 11 was given by Kokshaysky[26] as a function of mass for insects, birds, and piston-engine aircraft. The power laws for the available power for three groups of flyers in Fig. 11 are as follows.

Propeller/turboprop aircraft (22%):

$$P_A = 5.25W^{1.13} \tag{19}$$

Jet transports (12%):

$$P_A = 128.02W^{0.977} \tag{20}$$

Mean power for birds (41%):

$$P_{A,\mathrm{muscle}} = 3.4W^{0.9675} \tag{21}$$

Here the power is in watts and W is in newtons. Unlike the case for the required power, the power-law exponent for the power available is a free parameter because it is not necessarily constrained by the aerodynamic similarity.

The excess power $P_A - P_{\mathrm{mr}}$ is proportional to the rate of climb from the level cruising flight, that is, $R/C = (P_A - P_{\mathrm{mr}})/W$. For aircraft, there is no obvious upper limit for the weight to sustain level flight. For birds, however, there is a critical weight for $R/C = 0$ beyond which birds cannot sustain level flight by their muscle power From the scaling law $P_{\mathrm{mr,bird}} = 1.23W^{7/6}$ for the required cruise power and the empirical correlation $P_{A,\mathrm{muscle}} = 3.4W^{0.9675}$ for the available muscle power, we give 165 ± 102 N (16.8 ± 10.4 kg) as the upper limit of the bird weight for maintaining level flight. Here, the large error margins given based on the error propagation equation represent the statistical variation in a heterogeneous group of birds. This estimate is basically consistent with the collected data of the upper bound of the bird weight, as shown in Fig. 2. The largest flying bird (mute swan) has a mean weight of about 118 N (12 kg) and an upper bound of weight of about 151 N (15 kg). Note that the upper limit of the bird weight for takeoff should be lower because more power is needed. These results may shed light onto the long-debated problem of whether extinct pterosaurs[27–29] and archaeopteryx[30] could fly. The Cretaceous pterosaur pteranodon had a mass ranging from 12.9 to 29.8 kg ($W = 126$–292 N). According to the preceding estimate, if pteranodon muscle had the same power available as the mean bird muscle, the pteranodon could marginally sustain level flight as long as it jumped from cliffs, hills, or tall trees for takeoff. Archaeopteryx, the ancestor of modern birds, follows the same power laws in the wingspan and wing area as modern birds.[30] However, physiologists and paleontologists have argued that the flight muscle ratio of archaeopteryx could be as low as 50% of that of modern birds. If it is true, the estimated maximum weight for archaeopteryx to sustain level flight is about 5.1 ± 3 N (0.52 ± 0.3 kg), which is larger than the generally accepted archaeopteryx mass of 0.25 kg. This indicates that archaeopteryx could fly using its relatively weak muscle.

Engine and Muscle Power

We compare the muscular power output of birds with the power of engines for aircraft. Figure 12 shows the mechanical power output of aeroengines as a function of the weight of aeroengines, where data are collected from Ref. 13. For piston engines, including model airplane engines (22%), the power law is

$$P_{\mathrm{piston}} = 287.73W_{\mathrm{engine}}^{0.8616} \tag{22}$$

whereas the power law for turboprop engines (15%) is

$$P_{\mathrm{turbo}} = 353.2W_{\mathrm{engine}}^{1.007} \tag{23}$$

Figure 13 indicates the dependency of the engine weight on the total weight of aircraft. For piston, turboprop, and jet engines (21%), data can be fit by a single power law

$$W_{\mathrm{engine}} = 0.327W^{0.8944} \tag{24}$$

Greenewalt[6] studied the relationship between the bird muscle weight and total weight for the passeriforms, shorebirds, and ducks. The power law for the average weight of the large pectoral muscle of the passeriforms, shorebirds, and ducks together is

$$W_{\mathrm{muscle}} = 0.2363W^{0.9675} \tag{25}$$

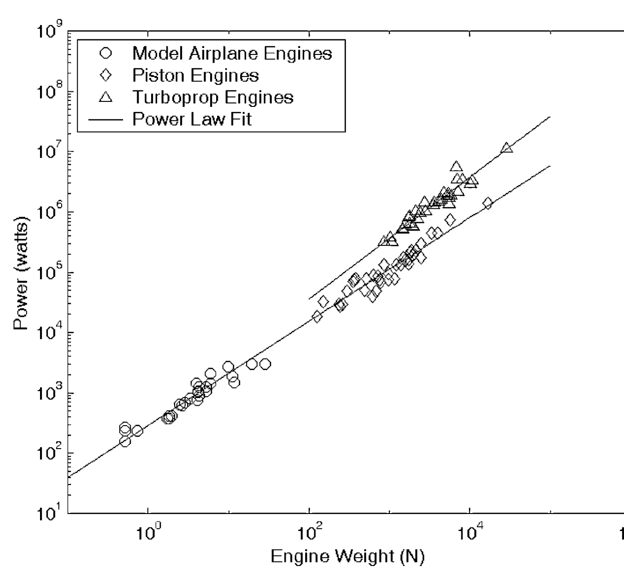

Fig. 12 Engine power output as function of engine weight.

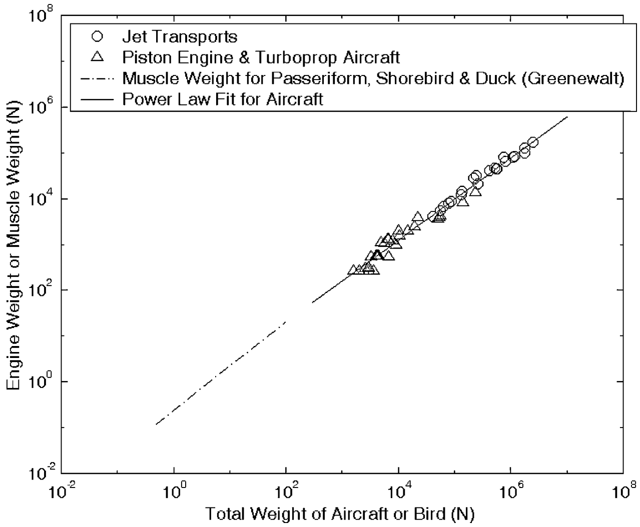

Fig. 13 Aeroengine and muscle weight as function of total weight of aircraft and birds.

McMasters[14] also gave a relation $W_{\text{muscle}} = 0.25W$ based on data for prey birds, passerine birds, and wading and web-footed birds. Interestingly, as shown in Fig. 13, Eq. (25) for the weight of the large pectoral muscle of birds is consistent with Eq. (24) for the engine weight of aircraft.

The mass-specific power of bird muscle given by Weis-Fogh[31] varies from 80 to 220 W/kg (8.16–22.45 W/N), and the average value is 141 W/kg (14.4 W/N), where the relative error is 41%. Thus, from Eq. (25), we have the mean bird muscle power available:

$$P_{A,\text{muscle}} = 3.4W^{0.9675} \tag{26}$$

Rayner[32] gave correlations based on collected data of measured metabolic power for 111 birds. If 20% of the metabolic power is converted to the mechanical power, a correlation for the mechanical power output of muscle in flight is $P_{A,\text{muscle}} = 1.7975W^{0.912}$, which is a lower estimate. Note that estimates of the available power of bird muscle are not conclusive, and a considerable variation exists among different data sources of metabolic power measurements in flight.[21] Other correlations for the available power of bird muscle include $P_{A,\text{muscle}} = 2.184W^{2/3}$ (Ref. 15) and $P_{A,\text{muscle}} = 1.921W^{0.74}$ (Ref. 19).

Next, we compare the mass-specific power (power density) of bird muscle with that of aeroengines, where the mass-specific power is defined as the mechanical power output per mass. Figure 14 gives a comparison of the mass-specific power between bird muscles and various aeroengines. Here, we use data of the mechanical power for birds, bats, and insects compiled by Weis-Fogh[31] and data for piston and turboprop engines from Ref. 13. For model airplane engines, we collect data from the specifications of Norvel, Irvine, and OS engines (www.sigmfg.com and www.osengines.com [cited 15 October 2002]). Evidently, the mass-specific power of bird muscles is much lower than that of aeroengines.

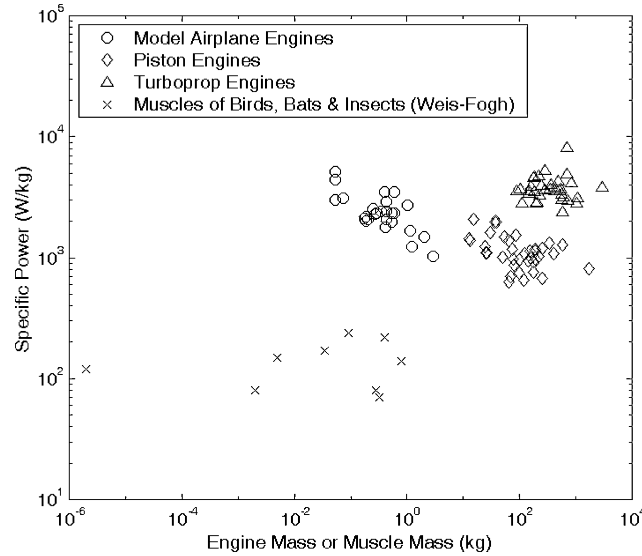

Fig. 14 Mass-specific power of muscles of birds and aeroengines as function of weight of engine or muscle.

III. Discussion

Estimated Maximum Range Power and Velocity

The time-area-averaged MST theory for flapping flight,[33] gives an induced relation for the power coefficient C_P as a function of the parameters C_W, $C_{D_{\text{para}}}$, and e_{flap}:

$$C_P = C_{D_{\text{para}}} + C_W^2 \big/ 4e_{\text{flap}} - C_W^{\beta} \big/ 4\alpha \tag{27}$$

where the coefficient α and exponent β are $\alpha(e_{\text{flap}}) = 55.83e_{\text{flap}}^2 - 26.27e_{\text{flap}} + 5.09$ and $\beta(e_{\text{flap}}) = -0.725e_{\text{flap}}^2 + 1.526e_{\text{flap}} + 2.84$, respectively. Here, the coefficients for the power, parasite drag, and weight are $C_P = P/(\rho A_b V^3/2)$, $C_{D_{\text{para}}} = D_{\text{para}}/(\rho V^2 A_b/2)$, and $C_W = W/(\rho V^2 A_b/2)$, respectively. The nondimensional empirical coefficient e_{flap} is the flapping span efficiency. The nominal definition of the flapping span efficiency e_{flap} is a ratio between the effective actuator disk area and the full circular area with a diameter b where $A_b = \pi b^2/4$. Physically speaking, the flapping span efficiency depends on not only the wing geometry, but also flapping kinematics. The first two terms on the right-hand side of Eq. (27) are simply the parasite (zero-lift) power and lift-induced power. The third term, which is empirical, can be considered as additional induced power that only has a limited effect at low speeds. Interestingly, in the time-area-average sense, the apparently different formulation in the MST theory for a flapping wing gives the almost same result for the required power as that from the classical lifting-line theory for a fixed wing. However, the flapping span efficiency e_{flap} in Eq. (27) has a different physical meaning from the Oswald span efficiency for a fixed wing. This point should be stressed when data of flapping flight are interpreted for comparison with fixed-wing aircraft.

The typical power curve is U shaped, and it has two characteristic velocities: the minimum power velocity V_{mp} and maximum range velocity V_{mr}. The maximum range velocity (the cruise velocity) V_{mr} corresponds to the tangential point of a line from the origin to the power curve, at which a migrating bird or aircraft covers the longest distance for a given amount of energy. The corresponding powers at V_{mp} and V_{mr} are denoted by P_{mp} and P_{mr}, respectively. There are the general relations $V_{\text{mr}} = 1.35V_{\text{mp}}$ and $P_{\text{mr}} = 1.146P_{\text{mp}}$. We consider a typical bird whose geometrical quantities obey the scaling laws $b = 0.506W^{1/3}$, $\bar{c} = 0.0646W^{1/3}$, $S_{\text{wing}} = 0.0327W^{2/3}$, and $AR = 7.83$. Adjusting the two free parameters $C_{D_{\text{para}}}$ and e_{flap}, we calculate the maximum range power and velocity as a function of W to fit both Greenewalt's data[6] for P_{mr} and Tennekes's data[12] for V_{mr} simultaneously. Thus, we obtain $e_{\text{flap}} = 0.5$ and $C_{D_{\text{para}}} = 0.0055$, and the corresponding correlations based on the calculations

$$P_{\text{mr}} = 0.9542W^{1.161} \tag{28}$$

$$V_{\text{mr}} = 9.067W^{0.162} \tag{29}$$

As shown in Fig. 10, Eq. (28) is compared well with Greenewalt's data[6] for birds and is close to Rayner's correlation $P_{\text{mr}} = 1.0564W^{1.161}$ given by the continuous vortex ring theory.[16] At the same time, as shown in Fig. 8, the maximum range velocity V_{mr} given by Eq. (29) is basically consistent with Tennekes's data and Rayner's correlation $V_{\text{mr}} = 8.0952W^{0.19}$. Here, the parasite drag coefficient $C_{D_{\text{para}}}$, which is a function of Reynolds number, is treated as a constant parameter in the time-area-average sense over a range of Reynolds numbers for birds. Therefore, if Tennekes's data and Greenewalt's data represent a statistical ensemble of data for a heterogeneous group of birds, $e_{flap} = 0.5$ and $C_{D_{para}} = 0.0055$ can be considered as the ensemble average values for birds.

Avian biologists have used the power–velocity relation for fixed-wing aircraft to estimate the Oswald span efficiency by fitting measured data for birds. Greenewalt[6] cited the measured values of the Oswald span efficiency, 0.7 and 0.51 for Coragyps atratus, 0.47 for Falco jugger, 0.21 for Columba livia, and 0.36 for Gyps africanus. Biologists were puzzled by the much lower span efficiency obtained for birds compared to the assumed values (0.8–0.9) previously adopted from the theory of fixed-wing aircraft in calculations for birds.[6] A reasonable explanation for these unexpected results cannot be extracted from the original meaning of the Oswald span efficiency that is the efficiency relative to the elliptical lift distribution. In fact, according to Eq. (27), the use of the power–velocity method is still legitimate for flapping flight, but the flapping span efficiency e_{flap} rather than the Oswald span efficiency is obtained. The flapping span efficiency has a different physical meaning, depending on not only the wing geometry (morphology), but also the flapping kinematics. The effective actuator disk area could be significantly smaller than the full circular area of a wingspan, and the value of e_{flap} for different birds might considerably vary. The lower flapping span efficiency indicates the higher induced drag associated with bird flight. Furthermore, it has been observed that the wing geometry (morphology) and flapping kinematics of birds vary with velocity. This implies that the flapping span efficiency e_{flap} may be velocity dependent, and, therefore, the power–velocity relation may deviate from a typical U-shaped curve.

Induced Drag

To estimate the induced-drag coefficient of birds in cruise flight based on the wing area $C_{D_{in},S_{wing}} = C^2_{W,S_{wing}}/(\pi AR e_{flap})$, we use the scaling laws $V_{mr} = 8.98W^{1/6}$, $S_{wing} = 0.0327W^{2/3}$, and $AR = 7.83$. For the air density of 1.21 kg/m^3, we have an estimate for the induced-drag coefficient of birds (54%) in cruise flight

$$C_{D_{in},S_{wing}} = 0.0193/e_{flap} \tag{30}$$

where the relative error is estimated using the error propagation equation. For $e_{flap} = 0.5$, the induced-drag coefficient for a typical bird is $C_{D_{in},S_{wing}} = 0.0386 \pm 0.0208$, which is comparable to the parasite drag coefficient of birds $C_{D_{para},S_{wing}} = 0.0344$. (See the next subsection.) Note that it is not easy to give the relative error for $C_{D_{para},S_{wing}}$. Thus, in a statistical sense of scaling, the total drag coefficient for birds is $C_{D,S_{wing}} = C_{D_{para},S_{wing}} + C_{D_{in},S_{wing}} = 0.073 \pm 0.0208$, where the error margins are underestimated because the error in $C_{D_{para},S_{wing}}$ is not included. The induced drag and parasite drag roughly equally contribute the total drag of birds.

Similarly, using the scaling laws for propeller/turboprop aircraft (52%) $V_{mr} = 15.52W^{1/6}$, $S_{wing} = 0.0262W^{2/3}$, and $AR = 8.15$, we have an estimate for the induced drag of aircraft:

$$C_{D_{in},S_{wing}} = 0.0032/e \tag{31}$$

where e is the Oswald span efficiency. For $e = 0.8$, the induced drag coefficient is $C_{D_{in},S_{wing}} = 0.004 \pm 0.0021$. At large Reynolds numbers in aircraft flight, the parasite drag coefficient of propeller/turboprop aircraft is $C_{D_{para},S_{wing}} = 0.02 - 0.044$. It can be seen that the induced drag coefficient is much smaller than the parasite drag coefficient in aircraft flight and it contributes 10–30% of the total drag coefficient of aircraft. The parasite drag is the dominant contributor to the total drag of aircraft. Furthermore, the induced drag coefficient of birds is much larger than that of aircraft. The additional induced drag associated with flapping, as pointed out by von Holst and Kuchemann[3] and Lighthill,[4] is due to nonlinear coupling between the flapping motion and unsteady induced velocity.

Parasite Drag

For comparison, the parasite drag coefficient $C_{D_{para}} = 0.0055$ for birds based on the disk area A_b is converted to the conventional drag coefficient $C_{D_{para},S_{wing}}$ based on the wing area S_{wing} by multiplying a factor $\pi AR/4$. For a typical bird with $AR = 7.83$, we have $C_{D_{para},S_{wing}} = 0.0344$, which is in the range of the parasite drag coefficient $C_{D_{para},S_{wing}} = 0.02 - 0.044$ for propeller/turboprop aircraft, but is much larger than $C_{D_{para},S_{wing}} = 0.013 - 0.02$ for jets.[34] Because no measured data exist for the parasite drag of a flapping wing, we cannot provide a rigorous comparison for the parasite drag of birds. Accurate calculation of the parasite drag for birds by solving the Navier–Stokes equation is very difficult due to not only the complicated geometry and kinematics of flapping wings and the surface patterns of feathers, but also a lack of reliable models for transition and turbulence in highly unsteady flows. Measurements of the drag of a flapping device are also difficult because the thrust and drag cannot be cleanly separated from the measured force in the freestream direction. Limited data for several gliding birds (where their wings were fixed) indicate $C_{D_{para},S_{wing}} = 0.01 - 0.024$ (Ref. 6). Although how the unsteady motion of a flapping wing affects the time-averaged parasite drag is an open question, a quasi-steady-state analysis is given to provide some clues for this problem.

The instantaneous parasite drag of a flapping wing projected on the flight direction is $D_{para} = 0.5\rho V(V^2 + V_f^2)^{1/2} S_{wet} C_f$, where C_f is the effective skin-friction drag coefficient (including both the skin-friction drag and form drag) and V_f is the local flapping velocity. Thus, the time-averaged drag coefficient is $\langle C_{D_{para},S_{wing}}\rangle_t = C_f(S_{wet}/S_{wing})\langle(1 + V_f^2/V^2)^{1/2}\rangle_t$, where $\langle\,\rangle_t$ is the time-average operator. For a simple harmonic flapping motion $V_f(t) = V_{f0}\cos(\omega t)$, time-averaging operation over a flapping cycle $\omega t = [0, 2\pi]$ leads to

$$\left\langle C_{D_{para},S_{wing}}\right\rangle_t = C_f(S_{wet}/S_{wing})(2/\pi)\left(1 + k_V^2\right)^{\frac{1}{2}} \times E\left[k_V^2\Big/\left(1 + k_V^2\right), \pi/2\right] \tag{32}$$

where $k_V = V_{f0}/V$ is the relative flapping velocity to the flight velocity and $E(x, \pi/2)$ is Legendre's complete normal elliptic integral of the second kind. The effect of flapping on the time-averaged parasite drag coefficient is represented by the factor containing the parameter k_V. When $k_V = 0$, the steady-state value $\langle C_{D_{para},S_{wing}}\rangle_t = C_f S_{wet}/S_{wing}$ is recovered.

To estimate $k_V = V_{f0}/V$ in cruising flight of birds, we use Pennycuick's correlation[35] for the wing beat frequency $f(\text{hertz}) = W^{3/8} b^{-23/24} S_{wing}^{-1/3} g^{1/8} \rho^{-3/8}$. Use of the power laws for b and S_{wing} yields $f(\text{hertz}) \approx 7.34W^{-1/6}$. Thus, an estimate of the flapping velocity at the wing tip is $V_{f0} = fb/2 \approx 1.872W^{1/6}$, and the parameter k_V for birds in cruising flight is $k_V = V_{f0}/V_{mr} \approx 0.226$. Because the factor $(2/\pi)(1 + k_V^2)^{1/2} E[k_V^2/(1 + k_V^2), \pi/2]$ is 1.0126, the effect of flapping on the time-averaged parasite drag in cruising flight is very small. For slow flight, however, this effect is more pronounced.

From Eq. (32), the difference of the parasite drag between birds and aircraft may be mainly related to the effective skin-friction coefficient C_f, which is a function of the Reynolds number. At a typical Reynolds number of 10^5 for birds, based on the collected data by Templin,[15] the effective skin-friction coefficient is $C_f = 0.01 - 0.02$. Accordingly, the estimated parasite drag coefficient for birds is $\langle C_{D_{para},S_{wing}}\rangle_t = 0.027 - 0.055$, where $S_{wet}/S_{wing} = 2.66$ is used for birds. For aircraft where the Reynolds number is larger than 10^6, the effective skin-friction coefficient no longer significantly decreases as the Reynolds number increases further, and it varies between $C_f = 0.002 - 0.005$. Therefore, for aircraft, the estimated parasite drag coefficient is $\langle C_{D_{para},S_{wing}}\rangle_t = 0.0087 - 0.022$, where $S_{wet}/S_{wing} = 4.35$ is used for aircraft. These estimates suggest that the larger parasite drag coefficient in bird flight is probably due to the effect of lower Reynolds numbers.

Propulsive Efficiency

Using the scaling laws $V_{mr} = 8.98W^{1/6}$, $S_{wing} = 0.0327W^{2/3}$, and $P_{mr,bird} = 1.23W^{7/6}$, we estimate the propulsive efficiency for bird (60%) cruise flight, that is,

$$\eta_{prop} = V_{mr} D/P_{mr} = 9.62\rho C_{D,S_{wing}} \tag{33}$$

where the relative error is estimated using the error propagation equation. For the air density $\rho = 1.21$ kg/m^3, we know $\eta_{prop} = 11.64 C_{D,S_{wing}}$. As estimated before, the mean total drag coefficient of

Table 1 Scaling laws and derived results for birds and aircraft[a]

Quantity	Birds	Propeller/turboprop aircraft	Jet transports
Upper limit or MTO, N	$W_{\text{upper,bird}} = 1.242W$	$W_{\text{MTO}} = 1.282W$	$W_{\text{MTO}} = 1.282W$
Wingspan b, m	$0.506W^{1/3}$	$0.462W^{1/3}$	$0.462W^{1/3}$
Wing area S_{wing} m^2	$0.0327W^{2/3}$	$0.0262W^{2/3}$	$0.0262W^{2/3}$
Mean chord $\bar{c}$ m	$0.0646W^{1/3}$	$0.0567W^{1/3}$	$0.0567W^{1/3}$
Aspect ratio	7.83	8.15	8.15
Body length l_{body} m	$0.143W^{1/3}$	$0.41W^{1/3}$	$0.41W^{1/3}$
Maximum body diameter d_0, m		$0.0481W^{1/3}$	$0.0481W^{1/3}$
Wet and wing area ratio	2.66	4.35	4.35
Body fineness ratio	2.97	8.52	8.52
Wing loading	$30.6W^{1/3}$	$53W^{1/3}$	$53W^{1/3}$
Reynolds number Re_c,	$4.3 \times 10^4 W^{1/2}$	$6.2 \times 10^4 W^{1/2}$	$10.1 \times 10^4 W^{1/2}$
Cruise speed V_{mr}, m/s	$8.98W^{1/6}$	$15.52W^{1/6}$	——
Cruise power, W	$P_{\text{mr,bird}} = 1.23W^{7/6}$	$P_{\text{mr,aircraft}} = 1.67W^{7/6}$	$P_{\text{mr,aircraft}} = 1.67W^{7/6}$
Power available, W	$P_{A,\text{muscle}} = 3.4W^{0.9675}$	$P_A = 5.25W^{1.13}$	$P_A = 128.02W^{0.977}$
Muscle and engine weight, N	$W_{\text{muscle}} = 0.2363W^{0.9675}$	$W_{\text{engine}} = 0.327W^{0.8944}$	$W_{\text{engine}} = 0.327W^{0.8944}$
Maximum L/D	9.1	11.9	——
Flapping or Oswald span efficiency	$e_{\text{flap}} = 0.5$	$e = 0.6 - 0.9$	$e = 0.6 - 0.9$
Parasite drag $C_{D_{\text{para}},S_{\text{wing}}}$,	0.0344	0.02–0.044	0.013–0.02
Induced drag $C_{D_{\text{in}},S_{\text{wing}}}$,	$0.0193/e_{\text{flap}}$	$0.0032/e$	——
Propulsive efficiency η_{prop}	$9.62\rho C_{D,S_{\text{wing}}}$	$29.32\rho C_{D,S_{\text{wing}}}$	——

[a]Mean relative errors associated with the scaling laws indicated in the text.

birds is $C_{D,S_{\text{wing}}} = C_{D_{\text{para}},S_{\text{wing}}} + C_{D_{\text{in}},S_{\text{wing}}} = 0.073$, and therefore, the propulsive efficiency of bird flight is $\eta_{\text{prop}} = 0.85 \pm 0.51$. The error margins indicate a large statistical variation in this estimate although the efficiency cannot exceed one in reality. The flapping propulsive efficiency is reasonably high even though the error margins are large for a heterogeneous group of birds. The effect of the air density on the propulsive efficiency can be clearly seen in Eq. (33). An unsteady lifting line theory gave the propulsive efficiency of 69–76% (Ref. 36), and a theoretical model based on prescribed wake vortex structures constrained by minimum induced power requirements gave the efficiency as high as 86% (Refs. 37 and 38). Also, the time-area-averaged momentum stream tube theory for flapping flight[33] gave the propulsive efficiency similar to that of propellers. These theories do not directly calculate the pressure and skin-friction distributions on a flapping wing; instead they estimate the propulsive efficiency and other aerodynamic quantities based on models of a lifting line, wake vortex structure, and momentum stream tube. However, force measurements in a wind tunnel[39] showed that the propulsive efficiency (about 28%) of a mechanical flapper was much lower than the predicted efficiency of about 58% by an unsteady panel code. Numerical calculations using the Reynolds-averaged Navier–Stokes code OVERFLOW with a moving grid generator indicated the unexpectedly low propulsive efficiency of simulated bat and gull wings with simple flapping kinematics ("Flapping Wing Aerodynamics; 2D and 3D CFD Simulations" by S. P. Pao, 2000, unpublished). The propulsive efficiency is 0.2–0.28 for laminar flows on the wings at the Reynolds number of 5×10^4 and is 0.4–0.5 for turbulent flows on the wings at the Reynolds number of 2×10^5. At this stage, the propulsive efficiency of a flapping wing has not been fully evaluated by comprehensive experiments and computations. In particular, we should examine the possibly favorable effect of the flexible trailing edges of avian wings on flapping propulsion.

Similarly, by using the scaling laws $V_{\text{mr}} = 15.52W^{1/6}$, $S_{\text{wing}} = 0.0262\ W^{2/3}$, and $P_{\text{mr,aircraft}} = 1.67W^{7/6}$ for propeller/turboprop aircraft (56%), we have an estimate for the propulsive efficiency

$$\eta_{\text{prop}} = V_{\text{mr}} D / P_{\text{mr}} = 29.32\rho C_{D,S_{\text{wing}}} \tag{34}$$

For the air density $\rho = 1.21$ kg/m^3, we know $\eta_{\text{prop}} = 35.48 C_{D,S_{\text{wing}}}$. Because the propulsive efficiency cannot exceed one, according to this correlation, an estimated range for the total drag coefficient of propeller/turboprop aircraft is $C_{D,S_{\text{wing}}} = 0.0124 - 0.044$ in the ensemble-average sense.

Further Comments on Scaling

Table 1 summarizes the scaling laws and derived results for birds and aircraft. All of the scaling laws presented in this paper are expressed as a power function of the mean weight. The foundation of this kind of scaling is a direct combination of the geometrical similarity manifested by the one-third-power relation $l \sim W^{1/3}$ and the aerodynamic similarity described by $W/S_{\text{wing}} = \rho V^2 C_L/2$. The representation of the scaling laws based on the weight is usually dimensional rather than the conventional nondimensional form derived from dimensional analysis for engineering and scientific applications. Therefore, it is legitimate to question the universality of these dimensional scaling laws. However, this approach is particularly popular in the biological community because the weight is a convenient quantity to use.[6,9] The general concept of scaling and self-similarity emphasizes the aspects of scale invariance in various physical phenomena.[40] In fact, the scaling based on the weight is just a special manifestation of the general nondimensional scaling under certain conditions, and thus, it enjoys certain universality.

The characteristic length and area of a three-dimensional body are related to the body volume by $l = a_l V_{\text{body}}^{1/3}$ and $S = a_S V_{\text{body}}^{2/3}$, respectively, where V_{body} is the body volume and a_l and a_S are nondimensional constants determined by the geometrical relations between the body shape and length and area, respectively. Furthermore, the body volume is related to the body weight by $V_{\text{body}} = W/(g\rho_{\text{body}})$, where g is the gravitational constant and ρ_{body} is the mean body density. Hence, the nondimensional relations for the characteristic length and area are, respectively,

$$\frac{l(g\rho_{\text{body}})^{\frac{1}{3}}}{a_l W^{\frac{1}{3}}} = 1 \tag{35}$$

$$\frac{S(g\rho_{\text{body}})^{\frac{2}{3}}}{a_S W^{\frac{2}{3}}} = 1 \tag{36}$$

As clearly indicated by Eqs. (35) and (36), the scaling laws $l \sim W^{1/3}$ and $S \sim W^{2/3}$ exist for a heterogeneous group of birds or aircraft only when the body density ρ_{body} is constant and a_l and a_S are constant for the whole group. The condition that a_l and a_S are constant represents the geometrical similarity. Furthermore, we have the nondimensional relations for the characteristic velocity and power, respectively,

$$\frac{V\sqrt{\rho C_L a_S}}{\sqrt{2}(g\rho_{\text{body}})^{\frac{1}{6}}W^{\frac{1}{6}}} = 1 \tag{37}$$

$$\frac{P(L/D)\sqrt{\rho C_L a_S}}{\sqrt{2}(g\rho_{\text{body}})^{\frac{1}{6}}W^{\frac{7}{6}}} = 1 \tag{38}$$

Besides the geometrical similarity and constant body density, establishing the scaling laws $V \sim W^{1/6}$ and $P \sim W^{7/6}$ for the velocity and power requires additional conditions: the constant air density ρ, constant lift coefficient C_L, and constant lift-to-drag ratio L/D. Clearly, these necessary conditions for the similarity cannot be exactly satisfied for a heterogeneous group of birds or aircraft, and the deviations from these conditions introduce the statistical errors and decrease the degree of correlation in regression. In particular, the scaling for the velocity is more susceptible to the variations because the functional dependency of the velocity to the weight is weaker.

IV. Conclusions

The geometrical quantities, cruise velocity, and cruise power of birds and aircraft can be approximately scaled as a single function of the mean weight. The scaling law for the upper bound of the bird weight is interestingly close to that for the MTO weight of aircraft. The scaling laws for the wingspan, wing area, and mean wing chord are roughly the same for birds and aircraft. The wing aspect ratios for birds and aircraft are 7.83 and 8.15, respectively. Nevertheless, the body length of birds is only about 35% of that of scaled-down aircraft. The ratios between the total wet area and wing area for birds and aircraft are 2.66 and 4.35, respectively. The body fineness ratios for birds and aircraft are 2.97 and 8.52, respectively. From a scaling point of view, the wing loading and cruise velocity of birds are only 58% of those of scaled-down aircraft. As a result, the bird weight is significantly larger than the weight of scaled-down aircraft at the same Reynolds number, indicating that a typical bird has to generate larger lift for cruising flight. The cruise power of birds is about 74% of that of scaled-down aircraft. The estimated maximum lift-to-drag ratios for birds and aircraft are 9.1 and 11.9, respectively. The scaling laws for the power available are given, providing interesting results such as the upper weight limit for bird flight when it is combined with the scaling laws for the required power. Furthermore, the scaling law for the muscle weight of birds is intriguingly close to that for the aeroengine weight.

The aerodynamic consequences of the scaling laws are explored based on simple theoretical models. The flapping span efficiency estimated for bird flight is 0.5, which is lower than the Oswald span efficiency (0.6–0.9) of typical aircraft, and therefore it indicates larger induced drag. The estimated induced-drag coefficient based on the wing area for birds is 0.0386 ± 0.0208, which is much larger than that of propeller/turboprop aircraft. Nonetheless, the estimated mean parasite drag coefficient based on the wing area for birds is 0.0344, which is comparable to that of propeller/turboprop aircraft. Hence, the mean total drag coefficient of birds is 0.073, which is contributed roughly equally by the induced drag and parasite drag, and is much larger than that of typical aircraft. The propulsive efficiency for bird cruise flight is 0.85 ± 0.51, where the large error margins represent the statistical variation in regression for a heterogeneous group of birds. In summary, birds and aircraft enjoy similarity in some important geometrical quantities. Compared with a scaled-down aircraft, a typical bird cruises at a lower speed with a reasonable propulsive efficiency and manages to overcome larger drag and generate larger lift to support its heavier weight.

References

[1] von Karman, T., and Burgers, J. M., "Problems of Non-Uniform and of Curvilinear Motion," *Aerodynamics Theory*, Vol. 2, edited by W. F. Durand, Julius Springer, Berlin, 1935, Chap. 5.

[2] Garrick, I. E., "Propulsion of a Flapping and Oscillating Airfoil," NACA Rept. 567, May 1936.

[3] von Holst, E., and Kuchemann, D., "Biological and Aerodynamical Problems of Animal Flight," *Journal of the Royal Aeronautical Society*, Vol. 46, No. 374, 1942, pp. 39–56.

[4] Lighthill, J., "Introduction to the Scaling of Aerial Locomotion," *Scale Effects in Animal Locomotion*, edited by T. J. Pedley, Academic Press, New York, 1977, pp. 365–404.

[5] Lighthill, J., *Mathematical Biofluiddynamics*, Society for Industrial and Applied Mathematics, Philadelphia, 1975, Chap. 8.

[6] Greenewalt, C. H., *The Flight of Birds*, American Philosophical Society, Philadelphia, 1972, pp. 1–67.

[7] Pennycuick, C., *Animal Flight*, Edward Arnold, London, 1972, Chap. 5.

[8] Pennycuick, C., *Bird Flight Performance*, Oxford Univ. Press, Oxford, 1989, Chap. 2.

[9] Pennycuick, C., *Newton Rules Biology*, Oxford Univ. Press, Oxford, 1992, Chap. 4.

[10] Shyy, W., Berg, M., and Ljungqvist, D., "Flapping and Flexible Wings for Biological and Micro Air Vehicles," *Progress in Aerospace Sciences*, Vol. 35, No. 5, 1999, pp. 455–506.

[11] Dunning, J. B., *CRC Handbook of Avian Body Masses*, Pt. 2, CRC Press, Boca Raton, FL, 1993.

[12] Tennekes, H., *The Simple Science of Flight*, MIT Press, Cambridge, MA, 1998, Appendix.

[13] Jackson, P., *Jane's All the World's Aircraft*, Jane's Information Group Ltd., Alexandria, VA, 2001.

[14] McMasters, J. H., "Reflections of a Paleoaerodynamicist," *Perspectives in Biology and Medicine*, Vol. 29, No. 3, Pt. 1, 1986, pp. 331–384.

[15] Templin, R. J., "The Spectrum of Animal Flight: Insects to Pterosaurs," *Progress in Aerospace Sciences*, Vol. 36, 2000, pp. 393–436.

[16] Rayner, J. M. V., "Thrust and Drag in Flying Birds: Applications to Birdlike Micro Air Vehicles," *Fixed and Flapping Wing Aerodynamics for Micro Air Vehicle Applications*, edited by T. J. Mueller, Vol. 195, Progress in Astronautics and Aeronautics, AIAA, Reston, VA, 2001, Chap. 11.

[17] Dial, K. P., Biewener, A. A., Tobalske, B. W., and Warrick, D. R., "Mechanical Power Output of Bird Flight," *Nature*, Vol. 390, Nov. 1997, pp. 67–70.

[18] Tucker, V. A., "Metabolism During Flight in the Laughing Gull, Larus atricilla," *American Journal of Physiology*, Vol. 222, No. 2, 1972, pp. 237–245.

[19] Butler, P. J., and Bishop, C. M., "Flight," *Sturkie's Avian Physiology*, 5th ed., edited by G. C. Whittow, Academic Press, New York, 1994, Chap. 15.

[20] Rothe, H. J., Biesel, W., and Nachtigall, W., "Pigeon Flight in Wind Tunnel," *Journal of Comparative Physiology B*, Vol. 157, 1987, pp. 99–109.

[21] Rayner, J. M. V., "Estimating Power Curves of Flying Vertebrates," *Journal of Experimental Biology*, Vol. 202, 1999, pp. 3449–3461.

[22] Grasmeyer, J. M., and Keennon, M., "Development of the Black Widow Micro Air Vehicle," *Fixed and Flapping Wing Aerodynamics for Micro Air Vehicle Applications*, edited by T. J. Mueller, Vol. 195, Progress in Astronautics and Aeronautics, AIAA, Reston, VA, 2001, Chap. 24.

[23] DeLaurier, J. D., and Harris, J. M., "A Study of Mechanical Flapping-Wing Flight," *Aeronautical Journal*, Oct. 1993, pp. 277–286.

[24] DeLaurier, J. D., "The Development and Testing of a Full-Scale Piloted Ornithopter," *Canadian Aeronautics and Space Journal*, Vol. 45, No. 2, 1999, pp. 72–82.

[25] Keennon, M. T., and Grasmeyer, J. M., "Development of Black Widow and Microbat MAVs and a Vision of the Future of MAV Design," AIAA Paper 2003-3327, 2003.

[26] Kokshaysky, N., "Some Scale Dependent Problems in Aerial Animal Locomotion," *Scale Effects in Animal Locomotion*, edited by T. J. Pedley, Academic Press, New York, 1977, pp. 421–435.

[27] Lawson, D. A., "Pterosaur from the Latest Cretaceous of West Texas: Discovery of the Largest Flying Creature," *Science*, Vol. 187, 1975, pp. 947, 948.

[28] Greenewalt, C. H., and Lawson, D. A., "Could Pterosaurs Fly?," *Science*, Vol. 188, 1975, pp. 676, 677.

[29] Stein, R. S., and McMasters, J. H., "Aerodynamics of Long Pterosaur Wing," *Science*, Vol. 191, 1976, pp. 898, 899.

[30] Shipman, P., *Taking Wing, Archaeopteryx and the Evolution of Bird Flight*, Simon and Schuster, New York, 1998, Chap. 11.

[31] Weis-Fogh, T., "Dimensional Analysis of Hovering Flight," *Scale Effects in Animal Locomotion*, edited by T. J. Pedley, Academic Press, New York, 1977, pp. 405–420.

[32] Rayner, J. M. V., "The Mechanics of Flight and Bird Migration Performance," *Bird Migration (Physiology and Ecophysiology)*, edited by E. Gwinner, Springer-Verlag, Berlin, 1990, pp. 283–299.

[33] Liu, T., "Time-Area-Averaged Momentum Stream Tube Theory for Flapping Flight," *AIAA Journal* (submitted for publication).

[34] McCormick, B. W., *Aerodynamics, Aeronautics and Flight Mechanics*, 2nd ed., Wiley, New York, 1995, Chap. 4.

[35]Pennycuick, C., "Wingbeat Frequency of Birds in Steady Cruising Flight: New Data and Improved Predictions," *Journal of Experimental Biology*, Vol. 199, 1996, pp. 1613–1618.

[36]Phlips, P. J., East, R. A., and Pratt, N. H., "An Unsteady Lifting Line Theory of Flapping Wings with Application to the Forward Flight of Birds, *Journal of Fluid Mechanics*, Vol. 112, 1981, pp. 97–125.

[37]Hall, K. C., and Hall, S. R., "Minimum Induced Power Requirements for Flapping Flight," *Journal of Fluid Mechanics*, Vol. 323, 1996, pp. 285–315.

[38]Hall, K. C., and Hall, S. R., "A Rational Engineering Analysis of the Efficiency of Flapping Flight," *Fixed and Flapping Wing Aerodynamics for Micro Air Vehicle Applications*, edited by T. J. Mueller, Vol. 195, Progress in Astronautics and Aeronautics, AIAA, Reston, VA, 2001, Chap. 13.

[39]Vest, M. S., and Katz, J., "Aerodynamic Study of a Flapping Wing Micro-UAV," AIAA Paper 99-0994, 1999.

[40]Barenblatt, G. I., *Scaling, Self-Similarity, and Intermediate Asymptotics*, Cambridge Univ. Press, Cambridge, England, U. K., 1996, Chap. 1.

K. Fujii
Associate Editor